

On string fields and superstring field theories

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

JHEP08(2009)044

(<http://iopscience.iop.org/1126-6708/2009/08/044>)

[The Table of Contents](#) and [more related content](#) is available

Download details:

IP Address: 80.92.225.132

The article was downloaded on 03/04/2010 at 10:21

Please note that [terms and conditions apply](#).

On string fields and superstring field theories

Michael Kroyter^{a,b}

^aCenter for Theoretical Physics, Massachusetts Institute of Technology,
Cambridge, MA 02139, U.S.A.

^bSchool of Physics and Astronomy, The Raymond and Beverly Sackler Faculty of Exact Sciences,
Tel Aviv University, Ramat Aviv, 69978, Israel

E-mail: mikroyt@tau.ac.il

ABSTRACT: We offer some thoughts regarding the space of string fields. We suggest that this space should be identified as the odd component of a star-algebra and focus among other issues on the role of the mid-point. We argue that theories with mid-point insertions in the action, such as the modified cubic theory can be well behaved, even if this mid-point insertion has a non-trivial kernel.

We then discuss the recent proposal by Berkovits and Siegel of a non-minimal superstring field theory. In this theory the action contains a mid-point insertion of a non-zero conformal weight. We show that, while this is a-priori a problem, it might be possible (in the NS sector) to make sense out of this theory by regularizing it. A cleaner resolution of the problem is to extend the non-minimal sector in a way that allows a zero-weight mid-point insertion with the desired properties. We also study the generalisation of the theory to the NS– sector and explain the problems with defining the Ramond sectors.

We show that the non-minimal theory supports all the known solutions of the standard modified cubic superstring field theory, including the GSO+ vacuum solution. The properties of the solutions carry over to the non-minimal theory. In particular, the vacuum solution has the correct tension and cohomology.

KEYWORDS: String Field Theory, Superstrings and Heterotic Strings

ARXIV EPRINT: [0905.1170](https://arxiv.org/abs/0905.1170)

Contents

1	Introduction	1
2	The space of string fields	3
3	Non-minimal superstring field theory	12
4	Classical solutions	18
5	An extended non-minimal formalism with a zero-weight insertion	20
6	Other sectors	22
7	Conclusions	23

1 Introduction

The first attempt at constructing a covariant open superstring field theory was made by Witten [1], following his construction of the bosonic theory [2].¹ The most salient feature of the RNS superstring as compared to the bosonic theory is the redundancy of vertex operators: Each state can be represented by infinitely many vertex operators that differ by a new quantum number called picture number [4]. CFT expectation values are non-zero only for -2 picture number. Witten suggested to work in the NS sector with string fields whose picture number is “the natural” -1 picture. This implied that a picture changing operator should be inserted on the interaction term. Associativity of the star-product, gauge symmetry of the theory as well as the global residual conformal symmetries generated by the K_n operators [1], imply that the picture changing operator has to be inserted at the string mid-point.

The string mid-point is invariant under the star product.² For Witten’s theory this implies that picture changing operators collide when one considers gauge transformations or string scattering [5]. These collisions render the theory inconsistent, since the OPE of the picture changing operator with itself has a double pole. A resolution to these problems was suggested by using a picture zero NS string field and an overall mid-point insertion (in the NS sector) of the double inverse picture changing operator Y_{-2} [6–8]. In the most

¹For a detailed introduction to string field theory and recent results within it one can consult [3]. Section 8 therein contains an introduction to superstring field theories.

²Throughout this work the star product is implicit. It should cause no confusion, as there is no other way to multiply string fields. Another convention we are using is writing $[\cdot, \cdot]$ for the graded-commutator.

common formulation this operator is built using the doubling trick as a bi-local operator, composed of two insertions of the inverse picture changing operator Y ,

$$Y_{-2} = Y(i)Y(-i), \quad Y(z) = c\partial\xi e^{-2\phi}(z). \quad (1.1)$$

While this theory does not suffer (in the NS sector) from collisions of picture changing operators in gauge transformations and in the evaluation of tree amplitudes, it was criticized for the use of the picture changing operators on the ground that they support non-trivial kernels. The problem with the kernels is that they (naively) imply that the equation of motion derived from the action is not equivalent to the desired one,³

$$Y_{-2}(QA + A^2) = 0 \quad \stackrel{?}{\iff} \quad QA + A^2 = 0. \quad (1.2)$$

The states in the kernel of Y_{-2} can be characterized by specific mid-point insertions. It is not clear if such states should be allowed as part of the space of string fields. On the other hand, the opposite is also not obvious. The lack of understanding of the space of string fields is one of the biggest open problems in the realm of string field theoretical research. It is also one of the most ignored ones. While we do not know how to solve this problem, there are several observations that can be made regarding this space. We discuss and speculate on this issue in section 2. The following are some of the conclusions we draw,

- The space of string fields is part of a star algebra.
- Finite superpositions of regular wedge states with local insertions in their interior are part of the space of string fields.
- Insertions on the boundary and especially at the mid-point are problematic and should be discarded in many cases, if not always.
- Mid-point insertions over string fields, if allowed at all, can only be of operators of zero conformal weight.
- Mid-point insertions in the definition of an action play a different (in some sense complementary) role than mid-point insertions over string fields.
- The space of string fields depend on the theory at hand.

From the discussion of 2, it seems reasonable that in the cubic theory the problematic mid-point insertions should not be allowed. Hence, we believe that the cubic theory can be trusted in the NS sector. Nonetheless, one cannot discard mid-point insertions when the Ramond sector is included, since the gauge transformations then include explicit mid-point insertions [6]. These insertions do not belong to the kernel of Y_{-2} , but they lead to divergences that render the theory inconsistent [11]. One may still consider the theory, claiming that it has no gauge symmetry, but we believe that this theory would not be

³Another formulation of superstring field theory was given by Berkovits [9]. This formalism avoids picture-changing operators altogether and for this reason is often considered as a more reliable one. Nonetheless, it was recently found that, up to some subtleties, the two formalisms are classically equivalent [10, 11].

consistent at the quantum level and at any rate it cannot be the field theory of open superstrings.

Recently, Berkovits and Siegel suggested to extend by a non-minimal sector the space of operators on which the string field can depend [12]. From a first-quantized point of view this sector is trivial, since it decouples by the quartet mechanism [13]. It was then suggested to construct a string field theory which includes this non-minimal sector, where the Y_{-2} insertion is replaced by a measure factor \mathcal{N}_ρ , with ρ being an arbitrary parameter. This factor has no kernel and its functional integral over the non-minimal sector reduces to Y_{-2} , regardless of the value of ρ . We give more details of this construction in section 3, where we see that the non-zero conformal weight of \mathcal{N}_ρ poses some difficulties. We further demonstrate that despite that the theory might still make sense (in the NS sector).

The motivation of the construction [12] was the problem with the kernel of Y_{-2} . As we argue in section 2, this is not really a problem. In any case, if one adopts another point of view regarding the space of string fields, one should be aware of the fact that in addition to the potential problems with the kernel of a mid-point insertion there are potential problems with the space of operators whose OPE with the insertion is singular. Hence, a formulation with a regular mid-point insertion, such as [12, 14] is not different conceptually from the one with the Y_{-2} insertion.

In section 4, we further check the non-minimal theory by studying the counterparts of known analytical solutions of the minimal theory. All the solutions of the minimal theory are automatically solutions of the non-minimal one. We illustrate that the “tachyonless tachyon solution” has the same properties in the non-minimal theory as in the minimal one.

In section 5, we return to the problem of a non-zero-weight insertion and offer for it another, more trustworthy resolution. This resolution is based on further extending the space of conformal fields on which the string field depends by another quartet. We choose this quartet in a way that avoids the introduction of new zero modes. Then, we modify the mid-point insertion, including in it dependence on the new quartet variables. The new mid-point insertion still reduces to Y_{-2} upon integration over the complete set of non-minimal variables, while having a zero conformal weight.

The extension of the non-minimal theories to the NS– and to the Ramond sectors is studied in section 6. We find out that while there is no problem with defining the NS– sector, the extension of the formalism to the Ramond sectors is not trivial and is bound in any case to suffer from singularities in its finite gauge transformations, analogous to those of the minimal theory. We conclude with some closing remarks in section 7.

2 The space of string fields

Here, we present some thoughts regarding the form of the space of string fields.⁴ These are preliminary ideas that try to sum up our knowledge of the form of known solutions and other legitimate string fields, as well as the basic algebraic structures and the restrictions that they pose. While this problem is largely ignored, it is nevertheless important and is relevant

⁴There are several types of “string fields”, e.g., the “physical” string fields that appear in the action and the gauge string fields. Here, we refer to the physical string fields simply as string fields.

also in the context of the recent development that followed Schnabl’s solution [15]. In particular, the requirement that the equations of motion are obeyed even when contracted with the solution itself and the role of the “phantom piece” at the limit $n \rightarrow \infty$ in this respect [16, 17] are important for proving Sen’s first conjecture [18, 19]. Also, the evaluation of the cohomology around the solution specifically assumed particular behaviour of the allowed string fields [20].

The exact definition of the space of string fields is one of the biggest unknowns in string field theory. While it is common to use terms such as “Hilbert space” in this context, it is not clear if the desired space is a Hilbert space, since the natural inner product of string fields is not positive definite, due to the existence of the ghosts.⁵ One may think that it is possible to use the basis elements of the “Hilbert space” without any constraints on the coefficients. This is not the case. To see that consider the insertion

$$\mathcal{O}(z) |0\rangle = \sum_{n=0}^{\infty} \frac{z^n}{n!} \mathcal{O}^{(n)}(0) |0\rangle. \tag{2.1}$$

If there were no restrictions on the coefficients, such a state with $|z| > 1$ (in the upper half plane coordinates) could have been produced. This would result in an insertion of the operator \mathcal{O} not in the interior, but at the local coordinate patch reserved for the test string field with which this one is to be contracted. The test string field can be an arbitrary element of the space of string fields and in particular it might have an operator insertion at this point, whose OPE with \mathcal{O} is singular. Hence, the insertion at $|z| > 1$ is bound to produce singularities in the star product. The star product of any two elements of the space of string fields should be well defined in order for the action to make sense. Hence, states of the form (2.1) with $|z| > 1$ should be discarded. We conclude that the linear combinations should be somehow constrained, despite the absence of a natural (positive definite) norm on the “Hilbert space”.

Local insertions at $|z| < 1$ should probably be allowed. What about the case $|z| = 1$? Multiplying a string field containing a local insertion \mathcal{O}_1 at $e^{i\theta}$ by another string field, with an insertion \mathcal{O}_2 at $e^{i(\pi-\theta)}$ results in the collision of the operators \mathcal{O}_i . This product is singular, unless the two operators have a regular OPE. A possible resolution of this specific issue might be to “divide the boundary between the state we consider and the test state”, allowing operator insertions only at $\theta < \frac{\pi}{2}$ or at $\theta > \frac{\pi}{2}$. This prescription is asymmetric and is hence, inconsistent with the reality condition of the string field. The reality condition implies that if an insertion is allowed at $e^{i\theta}$, it should also be allowed at $e^{i(\pi-\theta)}$. An immediate corollary is that a minimal requirement from a local insertion on the boundary is to have a regular OPE with itself. One can still speculate that the range $0 < \theta < \frac{\pi}{2}$ can be divided to segments, with different insertions, whose mutual OPE’s are singular, allowed at different segments. While this construction naively allows for the

⁵One might choose, following [21], an arbitrary “reasonable” norm for defining the space of string fields and hope that the (topological) space it defines does not depend on the particular choice of this norm. Topology is indeed the required structure, since this is all that is needed in order to define convergence and limits. Nonetheless, we do not believe that this is a sensible way to define it, since it is not clear to us that any “natural norm” is sensible. See also the discussion below on separating the matter and ghost sectors.

coexistence of mutually inconsistent insertions, it does not respect the global symmetries generated by⁶

$$K_n \equiv L_n - (-1)^n L_{-n}. \tag{2.2}$$

Hence, it seems more reasonable to disregard these unnatural splittings. It can be seen from this discussion that the mid-point should be treated differently than the other points with $|z = 1|$, since it is invariant under the star product and is not related to the other points by the global K_n symmetries. We return to this issue below.

A very natural expectation is that the space of string fields is contained in a star-algebra. A possible objection to this point of view may rely on the fact that (for the cubic string field theories such as [2]⁷) the string field is classically restricted to be of ghost number one. The set of ghost number one string fields is not closed under the star product. Moreover, in the action and in the equation of motion only one or two star products appear. Hence, the requirement of forming a star algebra is not justified. We oppose this line of argument for several reasons:

- While classically the string field is indeed of ghost number one, quantum mechanically its ghost number is not restricted [23–25].
- The space of gauge string fields is (classically) of ghost number zero. Gauge transformations can be iterated. Hence, the space of (finite) gauge string fields is certainly closed.
- The star product is the primary algebraic entity of all the (open) string field theories, regardless of their polynomiality properties or ghost number restrictions. It seems natural to exploit the algebraic structure that is formed by the star product for defining the space of string fields.⁸

Note, however, that even in the quantum theory, where the ghost number is not restricted, the space of string fields differs from the algebra of gauge string fields. While the later is obviously a star-algebra, the former is obviously not one. The reason is that the space of string fields is odd. Hence, a product of two string fields is not itself a string field, since it is even. The gauge string fields can form a star algebra, since they are always even. The parity is always the same in both cases, regardless of the ghost number. This is achieved by working with odd coefficient fields. The coefficient fields are odd for a string field at an even ghost number and for a gauge string field at an odd ghost number. We suggest that the algebra one should consider consists of string fields at an arbitrary ghost number and whose parity is arbitrary. The even sub-algebra of this algebra forms the (quantum)

⁶These “residual Virasoro symmetries” were introduced by Witten in [1]. It was shown by Hata (and reported in the appendix of [22]), that on solutions these global symmetries reduce to gauge symmetries.

⁷Indeed, our line of argument is even stronger for the case of the non-polynomial theory [9], in which the string field carries zero ghost (and picture) numbers.

⁸One can consider the even more restrictive possibility, that the space of string fields forms a C^* -algebra. If this is indeed the case, it might be possible to exploit this structure for gaining a better understanding of the space of string fields [26].

space of gauge string fields, while the odd part of this space forms the sought after space of string fields. Both these spaces can be further restricted by imposing reality conditions.

The other algebraic structures that we have other than the star product are the integral and the derivation Q (BRST charge). We should demand that our space is invariant under the action of Q and that integrating any of its elements gives a finite result (i.e., we do not want to consider string fields with diverging action). Further, we assume that an identity element exists within this algebra. While it is clear that the identity string field exists as a functional over the space of string fields, as it implements the one-vertex, it is also known that this string field is of a somewhat singular nature. This stems exactly from its role as the one-vertex: It is a state of zero width, constructed for implementing a delta-functional gluing. Because of this nature, attempts to construct solutions based on the identity string field [27–30] were not successful in reproducing a calculable action, although some recent results in favour of these solutions were given in [31]. Insertions on the identity are always problematic, since in its geometric picture, the identity string field has no width. Hence, an insertion on the identity is necessarily an insertion on the boundary, which should always be handled with care. Moreover, the fact that the identity string field does not have a local coordinate patch in its interior (since it has no interior), results in some paradoxes, when insertions on it are considered [32].

An illustration for the problematic role of insertions on the identity can be seen as following. By allowing insertions on the identity one can write the quadratic term around a solution as a commutator. Moreover, one can eliminate the quadratic term altogether [33]. To that end we introduce (in the cylinder coordinates) the string field,⁹

$$J = \int_{i\infty}^{-i\infty} \frac{dz}{2\pi i} J_B(z) |\mathcal{I}\rangle, \tag{2.3}$$

obeying

$$J^2 = 0. \tag{2.4}$$

By definition we can write

$$QA = [J, A]. \tag{2.5}$$

It is clear that the kinetic term around a solution can now be written as

$$Q\tilde{A} = Q\tilde{A} + [A, \tilde{A}] = [A + J, \tilde{A}]. \tag{2.6}$$

The action can be written as

$$S = - \int \left(\frac{1}{2} A[J, A] + \frac{1}{3} A^3 \right) = -\frac{1}{3} \int (A + J)^3. \tag{2.7}$$

From (2.6) one can read that the solution

$$A = -J, \tag{2.8}$$

corresponds to a zero kinetic term and hence also to a trivial cohomology. On the other hand, (2.7) implies that the action of this solution is zero, in contradiction with Sen's

⁹The identity based solutions are generalizations of this construction.

conjecture. Hence, while it may be useful at times to use (2.5) for performing calculations, J of (2.3) should not be considered a genuine string field.

Regardless of the issue of legitimacy of states formed by insertions on the identity string field, we may add a formal identity to the algebra, which is needed for defining the finite gauge transformations. It seems to us, that when one prohibits insertions on the identity string field, there is no problem with identifying the formal identity with the identity string field. Note, however, that inconsistencies may arise with derivations of the star product that do not annihilate the identity. Such derivations should be discarded when the identity string field is considered. Luckily, the indispensable derivation Q , as well as η_0 in the supersymmetric case, annihilate the identity.

To summarize the discussion so far, we are looking for a space that forms a star-algebra with an identity, i.e., within this space all the relevant axioms should be obeyed, e.g., there are no associativity anomalies [34, 35], Q acts on elements of this space as a derivation, etc. . .

One may at this stage wonder whether a set obeying all the above exists at all. The answer to this question is affirmative. A trivial example would be the one-dimensional linear space spanned by the identity element. This is obviously not enough, since we should allow the unintegrated vertex operators acting on the vacuum in our algebra. The minimal algebra containing these string fields is the finite span of integer wedge state [32, 36–38] (W_n for $n \in \mathbb{N}_0$, with $W_0 = |\mathcal{I}\rangle$ and $W_1 = |0\rangle$) with insertions of unintegrated vertex operators at the integer points within the wedge state. An obvious generalization is the space \mathcal{A}_W spanned by a finite linear combinations of arbitrary finite wedge states (W_n for $0 \leq n < \infty$), with a finite number of local insertions in the interior. The space \mathcal{A}_W seems like a step towards the correct direction, but it also cannot be the final answer, since it neither contains the tachyon vacuum [15], nor is it closed under finite gauge transformations.

It is natural to assume that the algebra of string fields is maximal, i.e., it cannot be further extended. This requirement might also be important for being able to apply a variant of the fundamental lemma of the calculus of variations, without which it is impossible to derive the equations of motion from the action. A maximal space extending \mathcal{A}_W exists. To prove that consider the set of all (legitimate¹⁰) extensions of \mathcal{A}_W . This is a partially ordered set, with set inclusion defining the ordering, i.e.,

$$\mathcal{A}_1 \leq \mathcal{A}_2 \iff \mathcal{A}_1 \subseteq \mathcal{A}_2. \tag{2.9}$$

The fact that the partial ordering is given by set inclusion implies that every chain has an upper bound. Hence, by Zorn’s lemma a maximal extension exists, as stated.

While we can use Zorn’s lemma in order to prove that a maximal extension exists, this form of a proof is as far away from a constructive argument as it gets. What we would really

¹⁰One should define more accurately this “set of all possible extensions”, or it would neither be a set, nor would it be well defined. It is clear that we should only have as basis elements the ones that we already introduced. What is still needed is a notion of topology, in order to define convergence and equivalence classes of infinite sums of these elements. The ordering one should consider then should take into account both the elements of the space, as we suggest here, as well as the topology of the space. The ordering of topologies can again be achieved by set inclusion, so Zorn’s lemma can still be used. Here, we ignore this issue for simplicity.

like to have is a clear criterion to apply to string fields, in order to determine whether they are legitimate ones or not. Moreover, Zorn's lemma proves the existence of the extension, but not the uniqueness thereof. In fact, the maximal extension is not unique and further physical data should be used in order to decide among the possible maximal extensions. In order to demonstrate the non-uniqueness, let us return to the role of mid-point insertions. Such insertions, if allowed at all, should only be of operators of zero conformal weight. This fact can be understood in the unit disk coordinates. In this coordinates the conformal transformation used in the definition of the star product is

$$\zeta \rightarrow \zeta^{2/3}, \tag{2.10}$$

while the string mid-point is at $\zeta = 0$. The conformal transformation of the mid-point gives a proportionality factor of

$$\left(\frac{2}{3}\epsilon^{-1/3}\right)^h \xrightarrow{\epsilon \rightarrow 0} \begin{cases} \infty & h > 0 \\ 0 & h < 0 \\ 1 & h = 0 \end{cases}. \tag{2.11}$$

The case $h > 0$ should be excluded, since it would result in a divergent star product with all the Fock space states, which are certainly legitimate ones. For negative conformal weight, the star product of such string fields with other string fields is zero. While this is not a severe problem as the other case, it seems that the inclusion of such states will invalidate the derivation of the equations of motion from the action, since no generalization of the fundamental lemma of calculus of variations can exist for such states.¹¹

Examples of operators with $h = 0$, which can a-priori be inserted at $z = i$ on string fields in \mathcal{A}_W , include among others the operators,¹²

$$\mathcal{O}_1 = \xi, \quad \mathcal{O}_2 = \eta c. \tag{2.12}$$

Each of these operators has regular (in fact zero) OPE's with itself. Thus, each one of them can be included in an extension of \mathcal{A}_W ($\mathcal{A}_W \oplus \xi \mathcal{A}_W$ for example is a legitimate, non-maximal algebra). Nevertheless, we cannot include both, since the OPE $\mathcal{O}_1 \mathcal{O}_2$ is singular. This proves our assertion that the maximal extension of the algebra is not unique. Similar considerations regarding OPE's apply also for operator insertions at other points of the

¹¹A non-zero and non-singular result could presumably be obtained from the product of two string fields with $h < 0$ mid-point insertions, provided that some of the OPE's of the insertions with themselves have exactly the needed singularities for balancing the zeros of the conformal transformations. One then would have to verify that the resulting string fields still have regular OPE's with the themselves and with the former ones and so on. One could then allow arbitrary regular insertions in the interior. We still feel that such string fields should be ruled out, since then a particular insertion would be allowed only on a particular wedge, which seems unnatural.

¹²Note that the picture changing operators, whose conformal weight is also zero cannot be included, since they have singular OPE's with themselves. Since the mid-point is invariant under the star product, star multiplying two states with the same picture changing operator insertion at the mid-point (say, squaring a given string field containing an X insertion) will result in singularities of the star algebra, which we do not allow.

boundary. On the other hand, there are no restrictions on the conformal weight for non-mid-point boundary operator insertions.

More restrictions on the type of allowed string fields can come from specific properties of a given string field theory. The non-polynomial superstring field [9] theory contains in addition to Q also η_0 as a derivation. The algebra should be closed and non-anomalous (i.e., obey Leibniz' law) with respect to η_0 as well. The modified cubic superstring field theory [6–8] contains a mid-point insertion of the picture changing operator Y . Hence, for this theory the operator \mathcal{O}_1 of (2.12), whose OPE with Y vanishes, may be considered as a legitimate insertion, while the operator \mathcal{O}_2 , whose OPE with Y diverges, may not.¹³ Note the differences between mid-point insertions on a string field and a mid-point insertion in the action, such as the Y_{-2} in the cubic theory:

- While the former has to have regular OPE with itself, there is no such restriction for the action-insertion, since it cannot be iterated.
- While the former has to have $h = 0$, the later might have a non-zero h , provided that the theory is thought of as being some sort of a limit. Again, this stems from the fact that there is only one such insertion. Thus, a formal zero or infinite constant multiplying it can fix the problems with the conformal weight for all string fields. This is not the most elegant construction, but it should not be ruled out automatically. This situation was encountered in the construction of vacuum string field theory [39–41]. In section 3, we deal exactly with this case in the study of the Berkovits-Siegel proposal [12].
- Allowing for a mid-point insertion in the action prohibits mid-point insertions in the space of string fields of operators whose OPE with the action-insertion is singular or zero. The problem with the operators in the kernel of the mid-point action-insertion is known and it was the main ground for criticizing the cubic theory of [6–8]. We do not think that this is a problem. Rather, as stated, it tells us that such states cannot be considered as part of the string field algebra, since, again, the fundamental lemma of calculus of variations would not hold for them. The problem with the operators whose OPE with the mid-point action-insertion is singular was mostly overlooked in the literature. Nevertheless, string fields with these insertions cannot be allowed, since their action is not well defined. In particular it means that in the space of string fields mid-point insertions should be restricted not only in theories with mid-point insertion with a local kernel, such as [6–8], but also in theories whose mid-point insertion has no kernel such as [12, 14]. One might now be worried about all theories with mid-point insertions, since it seems that different choices of the action-insertion give different restrictions on the space of string fields. We do not believe that this is a problem, since:

¹³Note that the fact that $\mathcal{O}_1 = \xi$ does not belong to the small Hilbert space does not imply that it cannot be used here. It only implies that it cannot be inserted arbitrarily. A string field of the form $(\xi(z) - \xi_0)|V\rangle$ that belongs to the small Hilbert space is an example of an allowed element in an extension of \mathcal{A}_W .

- As we already claimed, mid-point insertions, if allowed at all, should any way be constrained, and there is more than one way to do that.
- The mid-point insertions might be thought of as gauge equivalent to more benign ones, but being at the verge of the gauge orbit. If this interpretation holds, it is not important if the mid-point insertion itself really exists, or is just a singular limit, since the gauge orbit exists anyway.
- It is perfectly sensible to have the space of string fields depend on the particular theory one studies. We turn now to a somewhat different example that illustrates this point.

Consider a superstring field theory with an $R+$ sector. The NS sector should contain states which look like the star product of two $R+$ states, which in the most simple case take the form of $R+$ insertions on the (NS) vacuum. The NS state then looks like a bi-local $R+$ insertion on W_2 . One might deduce that such states are inherent in any description of the NS+ sector. Now, we may repeat the above, only with an $R-$ sector. It seems improbable that both NS+ states based on $R+$ and NS+ states based on $R-$ coexist, since they are not mutually local. This problem does not arise in the worldsheet description, since there the two insertions are inserted at the same place and one is left, after evaluating the OPE, with a genuine NS+ state. In string field theory on the other hand the insertions are at distinct points and it might well happen that after multiplying two NS+ states one is left with two $R\pm$ states, which are closer to each other than to any other insertion. There might be several resolutions to this difficulty. We believe that the correct one is to assume that the exact content of the NS+ sector depends on the theory at hand. The space of string fields is different for the NS+ sector when it is coupled to $R+$, to $R-$, or to NS-.

Ramond sector insertions are solitons from the perspective of the NS sector. It is a particular case of an allowed infinite linear combination of basis elements. We already gave another such example, that of finite gauge transformations. Yet another explicit case of infinite linear combinations arises when we consider the wedge states, which as stated already have to be part of the algebra. Wedge states are particular cases of surface states [37, 42, 43]. Surface states can be defined by requiring that the integral of their star product with any test state is given by the CFT expectation value of this test state on a particular surface.¹⁴ Hence, a surface state is characterized by a deformation of the canonical disk. Infinitesimal deformations are described by the action of negative Virasoro generators on the vacuum. Thus, a surface state, which is characterized by a finite deformation, has to have the form,

$$|S\rangle = e^{\sum_{n=2}^{\infty} v_n L_{-n}} |0\rangle, \tag{2.13}$$

¹⁴This might sound as a bad definition, since the expectation value in a boundary CFT does not depend on the specific disk on which it is evaluated, as they are all conformally equivalent. However, there are two conformal maps here. One that defines the surface and the other that defines the way the test state is inserted (or rather defines its local coordinate patch). One of these conformal maps can be set to be an arbitrary function, while the other defines the surface state.

where the v_n are the coefficients defining the surface state.¹⁵ Expanding the exponent in (2.13), we see that any surface state can be written as an infinite sum of local insertions. Thus, our \mathcal{A}_W is already a space that can be characterized by infinite linear combinations.

Extending \mathcal{A}_W so as to include also other surface states is a very natural direction. The simplest surface state algebra extending that of the wedge states is the star algebra formed from (hybrid) butterfly states and wedge states [41, 44–48]. The states of this algebra can be characterized by (generalized) Schwarz-Christoffel maps with angles, which are multiples of $\frac{\pi}{2}$, in the cylinder frame. Other algebras, naturally extending the two previous ones, can be defined using other basic angles. The space of finite sums of elements of these spaces (presumably with a finite number of local insertions) forms the algebra of (generalized) polygons [48]. This is still a “small” algebra as compared to the full algebra of surface states (with insertions).

Considering (2.13) for defining surface states, one should note that the deformation might not only create a new surface, but also reduce an existing part of the surface. The simplest example for that is the identity string field, in which all the surface other than the local coordinate patch, reserved for the test state, was eliminated. Going further to wedge states with $n < 0$ is possible as far as calculating the coefficients is concerned and there is no obvious problem with the form of these coefficients as a function of n . Nonetheless, it should be clear from the discussion above that these string fields should be ruled out. One might still hope to be able to see some clue for a singularity when expanding the coefficients.

Let us illustrate that it is impossible to decide whether a string field is legitimate or not by the behaviour of the coefficient of various fields as a function of the level. Consider the generalization of wedge states to “unbalanced wedge states” [38]. These are string fields which look like wedge states when restricted to the matter or to the ghost component. However, the wedge states are different in the two sectors. Writing the states is straightforward, due to the factorization,

$$L_n = L_n^{\text{mat}} + L_n^{\text{gh}}, \tag{2.14}$$

and to the fact that the Virasoro operators of different sectors commute,

$$[L_n^{\text{mat}}, L_m^{\text{gh}}] = 0, \quad \forall n, m. \tag{2.15}$$

It was shown that unbalanced wedge states lead to inconsistencies and should probably be discarded from the space of string fields [38]. The coefficients of an unbalanced wedge states cannot behave worse than those of the two genuine wedge states defining it. We see that behaviour as a function of the level cannot guide us, as stated. One might now think that it is possible to solve the problem simply by discarding unbalanced surface states from the algebra. This is probably too restrictive, since there are other unbalanced surface states that do not suffer from the problems of the unbalanced wedge states [49]. Also, with the insertions on the surface states it might be non-trivial to distinguish linear combinations

¹⁵For example, it was shown in [32], that the wedge states W_n have only even non-zero coefficients and the first non-vanishing one is (W_n here is $|n+1\rangle$ of [32]), $v_2 = -\frac{(n-1)(n+3)}{3(n+1)^2}$. Note that, like all the other coefficients, it vanishes for $n = 1$, since W_1 is the vacuum state.

of balanced and non-balanced surface states. There is, however, another important lesson that we should study from this example: In string field theory there is never a genuine matter-ghost factorization.

3 Non-minimal superstring field theory

Here, we recall the recent proposal of a non-minimal superstring field theory [12]. This formulation uses a mid-point insertion with a non-zero conformal weight. We describe this theory and suggest that the problems related to the non-zero conformal weight of the insertion might be resolved by defining the theory using a limiting procedure.

The standard cubic open superstring field theory is defined in the NS sector by the action [6–8],

$$S = - \int Y_{-2} \left(\frac{1}{2} AQA + \frac{1}{3} A^3 \right), \tag{3.1}$$

where A is an NS+ string field in the zero picture and Y_{-2} is given by (1.1). The equation of motion takes the form,

$$QA + A^2 = 0, \tag{3.2}$$

where, as explained in section 2, we assumed in the derivation that the string fields behave nicely.

In [12] it was suggested that the string field should depend also on the non-minimal set of fields, (u, v) and (r, s) . Of these fields the first pair is a $\beta\gamma$ system with $\lambda = -\frac{1}{2}$ and the second pair is a bc system with $\lambda = -\frac{1}{2}$. Hence,

$$h(u) = h(r) = -\frac{1}{2}, \quad h(v) = h(s) = \frac{3}{2}, \tag{3.3}$$

and the energy momentum tensor of the non-minimal sector is,

$$T_{\text{NM}} = \frac{3}{2} \partial u v + \frac{1}{2} u \partial v + \frac{3}{2} \partial r s + \frac{1}{2} r \partial s. \tag{3.4}$$

These fields are defined in the upper half plane (in the canonical representation). One then also adds the anti-holomorphic fields. Alternatively, one can use the doubling trick and define the original fields as holomorphic in the whole plane.¹⁶

The non-minimal fields decouple from the cohomology of the world sheet theory by adding to the BRST charge the term

$$Q_0 = \oint \frac{dz}{2\pi i} r v. \tag{3.5}$$

¹⁶One may wonder if the u zero modes introduce a notion of picture similarly to the case of the γ zero mode. Of course, one can define a picture associated with this variable and in some sense we are working with the zero picture in this sector. This would be particularly transparent if one would fermionize the u, v system. In any case, one should not worry about this new picture number, since, as we show next, the non-minimal sector is redundant, i.e., it does not change the cohomology.

In order for this term to have the standard ghost number we choose,¹⁷

$$gh(u) = -2, \quad gh(v) = 2, \quad gh(r) = -1, \quad gh(s) = 1. \quad (3.6)$$

The decoupling originates from the fact that using Q_0 , any closed field in the non-minimal sector can be written as an exact one. This is usually referred to as the “quartet mechanism”, where two conjugate bosonic modes and two conjugate fermionic modes cancel each other in the cohomology. Here we have conformal fields, but one can study their modes separately and the logic holds with minor modifications.

We should also add to the BRST charge a piece of the form cT_{NM} , so as to ensure the relation

$$T_{\text{NM}} = [Q_{\text{NM}}, b]. \quad (3.7)$$

This suggests that we should consider,

$$Q_{\text{NM}} \stackrel{?}{=} \oint \frac{dz}{2\pi i} (rv + cT_{\text{NM}}). \quad (3.8)$$

However, such an ad-hoc definition might spoil the quartet argument and even the nilpotency property of Q . Luckily, there exists a similarity transformation, generated by

$$R = \oint c \left(\frac{3}{2} s \partial u + \frac{1}{2} \partial s u \right), \quad (3.9)$$

that gives a result very close to the above,

$$Q \equiv Q_{\text{M}} + Q_{\text{NM}} = e^R (Q_{\text{M}} + Q_0) e^{-R} \equiv e^R \tilde{Q} e^{-R}. \quad (3.10)$$

The form of Q_{NM} defined by this transformation is almost identical to (3.8) and is given by,

$$Q_{\text{NM}} = \oint \frac{dz}{2\pi i} \left(rv + cT_{\text{NM}} + \gamma^2 \left(\frac{3}{2} s \partial u + \frac{1}{2} \partial s u \right) \right). \quad (3.11)$$

The existence of the similarity transformation implies that the total BRST charge is nilpotent and its cohomology coincides with that of the minimal (RNS) sector. Note, that the last term in (3.11) does not contribute to (3.7). Hence, the non-minimal conformal fields are indeed primaries, with conformal weights that are given by (3.3).

The quartet mechanism proves the equivalence of the worldsheet theories with and without the non-minimal sector. At the string field theoretical level one has to append to the Q transformation (3.10), also its analogous one for the string field,

$$A \rightarrow e^R A. \quad (3.12)$$

The transformation (3.12) is realized by contour integrals, which can be deformed. Hence, QA and the powers of A transform in the expected way. Other than that we also need the superstring field theory action [12]. The action should be constructed in a way that

¹⁷One can make here also other choices for the ghost charge. We choose this particular option, in order to have later $gh(\chi) = -1$.

saturates the zero modes. The number of zero modes of a conformal field of dimension h is given by,

$$\#_{ZM} = \begin{cases} 0 & h > 0 \\ -2h + 1 & h \leq 0 \end{cases} . \quad (3.13)$$

The zero modes have to be saturated. Hence, a non-zero expectation value with the field content we have takes the form,

$$\langle \delta^2(\gamma)\delta^2(u)c^3r^2 \rangle \neq 0. \quad (3.14)$$

The expectation value is non-zero and finite, regardless of the exact location of the insertions, as long as one keeps similar insertions away from each other.¹⁸

In light of the above, one can define a one-parameter family of actions as,

$$S_\rho = - \int \mathcal{N}_\rho \left(\frac{1}{2} AQA + \frac{1}{3} A^3 \right), \quad (3.15)$$

where ρ is a parameter. The invertible measure factor \mathcal{N}_ρ is defined as a product of insertions at $\pm i$,

$$\mathcal{N}_\rho = \mathcal{N}_\rho(i)\mathcal{N}_\rho(-i), \quad \mathcal{N}_\rho = e^{\rho[Q,\chi]}, \quad \chi = uc. \quad (3.16)$$

Substituting Q one gets,

$$\mathcal{N}_\rho = e^{\rho\left(rc+u(\gamma^2+\frac{3}{2}c\partial c)\right)}. \quad (3.17)$$

The mid-point insertions of the non-minimal theory possess a space of operators whose OPE's with the insertions diverge. Consider as an example the weight-zero primary field $s\partial se^{2\phi}$. This field has a (leading order) pole of order $4n$ in its OPE with $[Q,\chi]^n$. Allowing a string field carrying this operator as a mid-point insertion in our theory would lead to an essential singularity in the evaluation of the action, while without the mid-point insertion in the action, such string fields could have been considered, since all powers of this operator are well defined. Hence, it seems that this theory is not a-priori superior over the theory with the Y_{-2} insertion.

An important property of the string field A is that it is odd. At the level of the minimal theory this property is implemented by the GSO projection. Here, we have a new set of odd fields. If not constrained, these fields can potentially lead to an even string field. The simplest possible constraint would have been to impose an even number of r, s insertions in the definition of the string field. This would have been a bad choice, since while Q_0 exchanges the spaces containing even and odd number of these insertions, the other part of Q leaves both spaces invariant. A better choice is to modify the GSO projection such that it would also count the non-minimal fermions in its definition. Now, all parts of Q exchange the two spaces, the NS+ string field A can be consistently defined and all the axioms are obeyed.

¹⁸It is possible to relax this condition by replacing, say, c^3 by $\frac{c''c'c}{2}$. For the bosonic insertions we can allow the delta functions to go over each other by using normal ordering with the “fermionized” ξ, η, ϕ variables (and similarly for u) [4] (see also [50] for a discussion on the geometric meaning of the delta functions). Here, following [12], we stay with the original variables.

As a first test of the action (3.15), consider the case in which $\frac{1}{2}AQA + \frac{1}{3}A^3$ is independent on the non-minimal sector. In this case the measure factor can be integrated out to give the standard Y_{-2} insertion, regardless of the value of ρ . This results from the fact that, in the non-minimal sector, only the r and u zero modes should be integrated,

$$\begin{aligned} \langle \mathcal{N}_\rho \rangle_{NM} &= \int dr_{-\frac{1}{2}} dr_{\frac{1}{2}} du_{-\frac{1}{2}} du_{\frac{1}{2}} e^{\rho(rc+u(\gamma^2+\frac{3}{2}c\partial c))} e^{\rho(\bar{r}\bar{c}+\bar{u}(\bar{\gamma}^2+\frac{3}{2}\bar{c}\partial\bar{c}))} \\ &= \int dr d\bar{r} du d\bar{u} \rho^2 rc \bar{r}\bar{c} e^{\rho u \gamma^2} e^{\rho \bar{u} \bar{\gamma}^2} = c\bar{c} \delta(\gamma^2) \delta(\bar{\gamma}^2) = Y_{-2}. \end{aligned} \tag{3.18}$$

Here, we first wrote the expectation value in terms of integration over the (two bosonic and two fermionic) zero modes. Then, we changed the variables to r, \bar{r} and u, \bar{u} ,

$$r = r_{\frac{1}{2}} + zr_{-\frac{1}{2}}, \quad \bar{r} = r_{\frac{1}{2}} + \bar{z}r_{-\frac{1}{2}}, \quad u = u_{\frac{1}{2}} + zu_{-\frac{1}{2}}, \quad \bar{u} = u_{\frac{1}{2}} + \bar{z}u_{-\frac{1}{2}}, \tag{3.19}$$

with the Jacobian of the transformation canceling between the bosonic and fermionic modes and left only the terms that will not vanish after the r, \bar{r} integration. Performing the integrals lead to the desired result. This ρ independent result leads one to believe that the action with different ρ values are equivalent. Indeed, as $[Q, \chi]$ is exact, \mathcal{N}_ρ can be everywhere inserted, regardless of the value of ρ and without formally changing on-shell world sheet calculations.

While the action of string fields that do not depend on the non-minimal sector is the same in the minimal and non-minimal theories, solutions of the former are not necessarily solutions of the later, since the kinetic operator is different in the two theories. Nonetheless, when one first uses the similarity transformation (3.10) and (3.12), one gets to a theory with \tilde{Q} as the kinetic operator.¹⁹ Other than changing Q and A , the transformation also changes the mid-point insertion, since $[R, Q\chi] \neq 0$. The change in the mid-point insertion is easily found once we notice that

$$[R, \chi] = 0. \tag{3.20}$$

This implies that the change in $Q\chi$ results only from the transformation of Q (3.10). Thus, the only change in \mathcal{N}_ρ is that the coefficient of the $uc\partial c$ term is now unity. This term drops out from the evaluation of the expectation value at the non-minimal sector (3.18), which gives Y_{-2} as before. Hence, for this transformed theory we can deduce the following properties of string fields that do not depend on the non-minimal fields:

- Such string fields are solutions if and only if they are solutions of the minimal theory.
- These solutions have the same action in both theories.
- The cohomology around these solutions is the same in both theories, again, due to the quartet argument.

The only thing we should still prove in order to demonstrate the classical equivalence of these two theories is that any solution of the \tilde{Q} -theory is gauge equivalent to a solution

¹⁹Note, that we transform the theory, but assume that the underlying CFT does not change, e.g., the conformal weights of the various non-minimal fields remain the same.

that lives in the minimal sector. Let A be a solution in this theory and assume that it is sensible to decompose it as a sum of insertions over wedge states.²⁰ What we mean by that is that the elements A_n are insertions over W_n and the equation of motion of $A = \sum A_n$ decomposes as,

$$\sum_{m \leq n} \tilde{Q}A_m + \sum_{k+m \leq n} A_k A_m = 0, \quad \forall n. \quad (3.21)$$

From the discussion in section 2, it seems plausible to assume that

$$n_0 \equiv \inf (\{n | A_n \neq 0\}) > 0. \quad (3.22)$$

If this is the case then (3.22) reduces for n_0 to the linearized equation of motion,

$$\tilde{Q}A_{n_0} = 0. \quad (3.23)$$

The quartet argument then implies that it is possible to find a (possibly trivial) linearized gauge transformation that is built over W_{n_0} that sets the non-minimal part of A_{n_0} to zero. Consider now (3.22) for $2n_0$. While this equation contains a non-linear part, its non-minimal component is zero, since, when the wedge state decomposition makes sense and no boundary insertions are allowed, the star product, as well as \tilde{Q} , do not change the number of non-minimal insertions. Hence the equation for the non-minimal component of (3.22) at level $2n_0$ reads,

$$\sum_{m \leq 2n_0} \tilde{Q}A_m^{(NM)} = 0. \quad (3.24)$$

Using the quartet argument again, we can push the non-minimal insertions to $m \geq 4n_0$ and so on, all the way to infinity. While this construction is still not a proof that every non-minimal string field is gauge equivalent to a minimal one, it seems like a strong indication in this direction and thus, to the classical equivalence of the minimal theory and the non-minimal one, based on \tilde{Q} . Using again the transformation generated by R (3.9), we conclude that the minimal and non-minimal theories are probably classically equivalent.

In the discussion regarding equivalence we implicitly assumed that the non-minimal theory is well defined and that the equations of motion can be derived from its action. It is not at all obvious that this is indeed the case. The problems result from the fact that this theory has a mid-point insertion, which is not of conformal dimension zero. In fact, χ and $[Q, \chi]$ are primary fields of dimension $-\frac{3}{2}$. This renders the theory not well defined. Specifically, the conformal expectation value, which appears in the definition of the theory has different meanings on different choices of the canonical disk. Trying to evaluate the expression on a different disk will induce a non-trivial transformation on the definition of

²⁰Following section 2, we know that an arbitrary string field can be written as an insertion over the vacuum state. However, in this representation the decomposition we propose will not work. Nonetheless, all known analytical solutions have a natural decomposition of the sort proposed here. Moreover, we do not really have to insist on a wedge state based solution. Rather, we have to demand that the solution can be decomposed to a set of insertions over an indexed set of string fields that can be treated as independent in some way and whose index is additive under the star product, such that (3.22) holds. This is the case, for example with the generalizations of Schnabl's solution studied in [51, 52]. Also, the sum that we refer to can well be an integral, or a superposition of discrete and continuous states.

the theory, due to the non-vanishing conformal weight. Luckily we can easily evaluate this change, since not only $[Q, \chi]$ is a (weight $\frac{3}{2}$) primary, but also $[Q, \chi]^n$ is a (weight $\frac{3}{2}n$) primary.²¹ This implies that the whole role of the conformal transformation is to induce a rescaling of ρ by a factor of $f'^{\frac{3}{2}}$.

The rescaling of ρ might seem to be not that bad. However, it introduces several problems. First, while it is assumed that the superstring field theory (3.15) is independent of ρ , it is not a-priori clear that this is indeed the case, due to the fact that it is an off-shell formalism. To overcome this problem one should prove the (quantum) ρ -independence. Another problem is that one should be careful so as to define the kinetic and potential terms on suitable surfaces, since even if the theory is ρ -independent, it is certainly not the case for each of the two terms separately. A relative rescaling of the two parts of the action can be compensated by a rescaling of A . However, here we don't have a rescaling by a number, but by an operator insertion at the mid-point. As suggested in section 2, one should avoid string fields with mid-point insertions of non-zero conformal weight operators. Hence, we cannot compensate the ρ rescaling in the case at hand. Moreover, the conformal transformations one usually considers in the context of the star product tend to have a singular derivative at the string mid-point. Thus, the rescaling we mention actually sends ρ to zero or infinity. This situation implies that we should treat the theory with care, regularize it in order to move the insertion away from the mid-point and at the end take the limit in a way that produces sensible results.

The procedure of regularizing the theory can be criticized on the ground that the mid-point should probably not be thought of as a limit of the interior points, as far as, say, defining string fields is concerned. Moreover, associativity of the star product and gauge invariance are lost during regularization and are recovered only as the limit is attained. We ignore these subtleties for now, assuming that our manipulations can be somehow justified. We offer a cleaner resolution of the problem in section 5.

For the purpose of regularization we implement the mid-point insertion as a product with the string field $I_{\delta,\epsilon}(\rho)$, defined in the cylinder coordinates as the wedge state W_δ with insertions of \mathcal{N}_ρ at $\pm ih \equiv \pm \arctan(i(1-\epsilon))$. Removing the regularization amounts to sending both δ and ϵ to zero. If we consider string fields which are wedge states W_{n_i} with finite number of local insertions at z_i , associativity is approximately recovered when $n_i \gg \delta$ and $h \gg \Im(z_i)$. Up to "small" corrections coming from non-associativity, the equations of motion take the regularized form,

$$I_{\delta,\epsilon}(\rho)(QA + A^2) = 0. \tag{3.25}$$

If $h \gg \Im(z_i)$ or if there are no insertions on the boundary,²² one can safely take the limit $\delta \rightarrow 0$. One should not worry about the fact that this limit produces a non-legitimate string field, since it only appears in a specific way in the equation of motion and should not be considered as an element of the star algebra. Now, we can multiply the equation of motion from the left by $I_{\delta,\epsilon}(-\rho)$, so as to obtain the correct equation of motion (3.2).

²¹This stems from the fact that the fields of the non-minimal sector form first-order systems.

²²String fields with ($h = 0$) insertions at the mid-point or line integrals that go all the way to the mid-point (such as the B insertion below) should also be regularized.

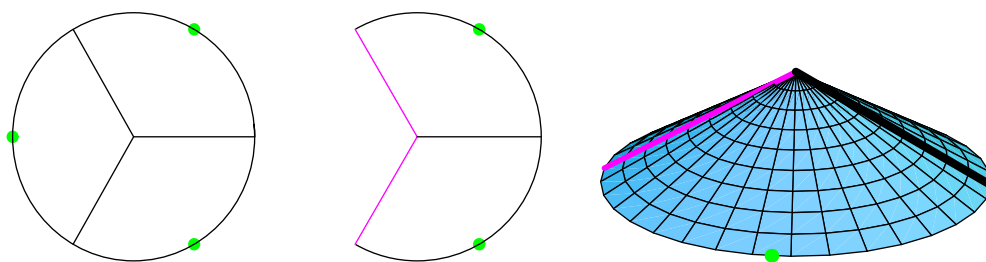


Figure 1. The three vertex (left), in its canonical conformal frame, with the insertions represented by green dots. The two vertex (middle) should be represented using the same mapping. The pink lines should be glued. One cannot use the canonical frame for the two vertex, since it would induce an infinite rescaling of ρ and hence change the mid-point insertions. A more “intuitive” picture (right) of the two-vertex is that of a cone (only one insertion is visible). The black and pink lines separating the two string fields are shown in this form as well. One could have decided to use the canonical surface for the two vertex, in which case the three vertex would have been inserted on a cone with a 3π circumference. The values of ρ in the two representations are formally related by an infinite rescaling.

Note that the limit $\epsilon \rightarrow 0$ ($h \rightarrow \infty$) is also necessary, in order to justify (3.25), since it is only in this limit that associativity, formally used in deriving this equation, is retained. Taking the same wedge width during regularization implies the geometric picture presented in figure 3. It seems that, at least classically and for this sort of solutions, the theory can be defined as a limit using this geometrical picture.

4 Classical solutions

In [53], Erler defined the vacuum solution of the modified cubic superstring field theory using its formal gauge representation [16].²³ For that, he simply took the same gauge string field as the one defining Schnabl’s tachyon vacuum in the bosonic theory [15],

$$\Lambda = Bc(0) |0\rangle, \tag{4.1}$$

where c is a c -ghost insertion in the cylinder coordinates and B is a line integral of the b -ghost in the same coordinate system,

$$B = \int_{i\infty}^{-i\infty} \frac{dz}{2\pi i} b(z). \tag{4.2}$$

The solution itself is then given by

$$A_M = Q_M \Lambda \frac{1}{1 - \Lambda}, \tag{4.3}$$

²³Despite the fact that this solution lives in the NS+ sector, it describes the vacuum without the original D-brane. This led to a proposal of a modified solution with support in the NS- sector, which might seem more adequate in the context of tachyon condensation [54]. It was claimed in [10] that these two solutions are gauge equivalent and the gauge family that interpolates between the two solutions was constructed. Hence, it seems natural to assume that Erler’s solution describes the non-perturbative vacuum also on the BPS D-brane, despite the absence of tachyons in this case.

where the subscripts “M” on A_M and Q_M remind us that they are the string field and BRST charge of the supersymmetric theory in the minimal sector (the same holds for K_M and T_M below). Substitution gives,²⁴

$$A_M = A_{\text{bos}} - W_{\frac{1}{2}} B \gamma^2 W_{\frac{1}{2}} = W_{\frac{1}{2}} c \frac{K_M B}{1 - W_1} c W_{\frac{1}{2}} - W_{\frac{1}{2}} B \gamma^2 W_{\frac{1}{2}}, \quad (4.4)$$

where A_{bos} is the bosonic solution and K_M is the line integral of the energy momentum tensor,

$$K_M = \int_{i\infty}^{-i\infty} \frac{dz}{2\pi i} T_M(z). \quad (4.5)$$

Actually, strictly speaking the first term in the r.h.s is not the same as the bosonic solution, the difference being that the later is defined using K_{bos} rather than K_M . Of course, K_M is not defined in the bosonic theory, but K_{bos} is defined in the RNS one (with a different central charge, due to the different dimensionality). Note, that the solution depends on K not only via its explicit presence, but also implicitly in defining the wedge states over which the solution is constructed,

$$W_n = e^{\frac{n\pi}{2} K} W_0. \quad (4.6)$$

We now want to identify the analogous solution in the non-minimal theory. One way would be to use the equivalence advocated in section 3. We prefer to follow another path for the construction of this solution. Both ways should result in solutions, which are at least gauge equivalent.

We again define the solution as a formal gauge solution using Λ (4.1), only now we have to include also Q_{NM} (3.11) in (4.3). This extra piece has no effect on the result other than changing K_M to $K \equiv K_M + K_{\text{NM}}$. Explicitly, the solution is,

$$A = W_{\frac{1}{2}} c \frac{K B}{1 - W_1} c W_{\frac{1}{2}} - W_{\frac{1}{2}} B \gamma^2 W_{\frac{1}{2}}, \quad (4.7)$$

Hence, we can say that the solution in this case is exactly the same as it is in the non-minimal theory, with the understanding that the geometric interpretation is uniform for all sectors. This is analogous to the declaration that the first term in (4.4) is “the same” as the bosonic solution. Note, that despite the discussion in section 2 regarding the problems with the unbalanced wedge states, there would have been no problem of principle in defining solutions over wedge states, which are not balanced between the minimal and non-minimal sectors, since the central charges of both sectors vanish separately.

For evaluating the action of the solution we have to regularize the insertion as stated. Also, in this case one has to regularize the B line integral for the same reason. However, it seems that this does not pose any problems. There is no explicit dependence on the non-minimal fields. Rather, they enter the evaluation of the action only via their presence in \mathcal{N}_ρ

²⁴We use the split-string notations here [55, 56], according to which operators are interpreted as if they are inserted (at the origin) on the identity string field. Then, all products are star products. Again, one should not worry about the legitimacy of the string fields that are built on the identity, which compose the solution, since the solutions itself is a legitimate string field.

and in the geometry of the surface on which the CFT expectation value is to be evaluated (K is geometrically realized as a derivative with respect to strip length). This allows us to evaluate the expectation value in the non-minimal sector first, leaving us exactly the Y_{-2} insertion. The fact that the same surface state is used in the minimal and the non-minimal sectors enables us to carry on with this Y_{-2} with the evaluation in the minimal sector. Now, we can use the fact that we are left with a zero-weight insertion and send δ and ϵ to zero. This results in exactly the action of Erler’s solution.

The evaluation of the cohomology around the solution is even simpler. We only have to note that the same contracting homotopy operator that works for Schnabl’s and Erler’s solutions [15, 53], works also here, where again, the non-minimal sector enters implicitly in the definition of the wedge states on which this string field is built. Since the action and the cohomology give the desired results, we conclude that the non-minimal theory also supports this “tachyonless tachyon solution”. This is in accord with our claim that the minimal and non-minimal theories are classically equivalent.

Similarly to the case of the non-perturbative vacuum one can translate other solutions to the non-minimal theory. In particular, the analytical solutions describing marginal deformations [57–65] can be recast in this language.

5 An extended non-minimal formalism with a zero-weight insertion

In section 3, we discussed the problems with the negative conformal dimension of χ . We suggested that if one thinks of the non-minimal superstring field theory as a limit in some sense of regularized theories, it might be possible to make sense out of it, since the formally infinite rescaling of ρ between different conformal frames becomes finite when the regularization is imposed. Nevertheless, it is desirable to find a more elegant resolution of the problem. A natural avenue to follow is to further extend the space of operators, in a way that would enable an $h = 0$ insertion.

The most straightforward extension is to declare that ρ itself is not a parameter, but a conformal field of dimension $h = \frac{3}{2}$. This cannot be the whole story, since if ρ is to be considered a genuine conformal field, it should contribute to the energy-momentum tensor and the total energy-momentum tensor should be reproduced by the relation

$$[Q, b] = T. \tag{5.1}$$

The appearance of ρ in Q implies that the cohomology might have changed. The simplest way to avoid that is to make ρ an element of a second quartet and define its contribution to Q in a way analogous to (3.11). This still cannot be the end of the story, since the field σ conjugate to ρ must have $h = -\frac{1}{2}$, which implies that it carries two zero modes. One could add a third quartet and eliminate the zero modes of the two new quartets by including a new piece in χ . However, now χ and $[Q, \chi]$ depend both on ρ and on σ . Hence, their product at the same point diverges. This can be resolved by including normal ordering in the definition, but even then, one does not get the desired result, as can be seen by a direct substitution.

While this modification does not work, there is still a way out. Let us add to the (u, v, r, s) quartet a second quartet $(\tilde{u}, \tilde{v}, \tilde{r}, \tilde{s})$, with all the fields having conformal dimension $h = \frac{1}{2}$. The new quartet has no zero modes. We now have,

$$Q_0 = \oint \frac{dz}{2\pi i} (rv + \tilde{r}\tilde{v}), \tag{5.2}$$

which we transform using a similarity transformation generated by,

$$R = \oint c \left(\frac{3}{2}s\partial u + \frac{1}{2}\partial su + \frac{1}{2}\tilde{s}\partial\tilde{u} - \frac{1}{2}\partial\tilde{s}\tilde{u} \right). \tag{5.3}$$

This leads to,

$$Q_{\text{NM}} = \oint \frac{dz}{2\pi i} \left(rv + \tilde{r}\tilde{v} + cT_{\text{NM}} + \gamma^2 \left(\frac{3}{2}s\partial u + \frac{1}{2}\partial su + \frac{1}{2}\tilde{s}\partial\tilde{u} - \frac{1}{2}\partial\tilde{s}\tilde{u} \right) \right), \tag{5.4}$$

where the energy momentum tensor is defined as usual for these fields,

$$T_{\text{NM}} = \frac{3}{2}\partial uv + \frac{1}{2}u\partial v + \frac{3}{2}\partial rs + \frac{1}{2}r\partial s + \frac{1}{2}\partial\tilde{u}\tilde{v} - \frac{1}{2}\tilde{u}\partial\tilde{v} + \frac{1}{2}\partial\tilde{r}\tilde{s} - \frac{1}{2}\tilde{r}\partial\tilde{s}. \tag{5.5}$$

Let us now define,

$$\chi = \tilde{u}^3 uc. \tag{5.6}$$

This operator is primary and has zero conformal weight. Moreover, its Q commutator and all the powers thereof are $h = 0$ primaries. Hence, there are no problems with a mid-point insertion of the operator,

$$\mathcal{N}_\rho = \mathcal{N}_\rho(i)\mathcal{N}_\rho(-i), \quad \mathcal{N}_\rho = e^{\rho[Q,\chi]} = e^{\rho(\tilde{u}^3(rc+u\gamma^2)+3\tilde{u}^2\tilde{r}uc)}. \tag{5.7}$$

We should also define the ghost numbers in a consistent way, e.g., demanding $gh(\chi) = -1$. The simplest way is to keep the ghost numbers introduced so far as before and define,

$$gh(\tilde{u}) = gh(\tilde{v}) = 0, \quad gh(\tilde{r}) = 1, \quad gh(\tilde{s}) = -1. \tag{5.8}$$

Integrating the (u, v, r, s) sector leads, as before, exactly to Y_{-2} , regardless of ρ 's value. Note, that the last term of $[Q, \chi]$ does not contribute, since it carries no r zero mode, but does carry a factor of c . As no dependence on the $(\tilde{u}, \tilde{v}, \tilde{r}, \tilde{s})$ sector remains, it can now be trivially integrated, since it carries no zero modes.

Solutions of the theory based on these two quartets can be defined as before. For the same reasons as in the one-quartet theory, they will share the properties of their ‘‘minimal’’ counterparts. However, unlike in the one-quartet case, the $h(\chi) \neq 0$ related difficulties are absent, there is no need to regularize the theory as part of its definition and there are no obstacles with more general solutions for which the regularization can potentially break down.

6 Other sectors

In this section, we discuss the incorporation of other possible sectors into the non-minimal theory. The NS− sector was first incorporated into superstring field theory in the context of the non-polynomial theory [66, 67]. There, it was understood that in order to keep the algebraic axioms of the star product intact, one should tensor the NS± string fields with the Pauli-matrices $\sigma_{0,1}$ respectively ($\sigma_0 \equiv \mathbf{1}_{2 \times 2}$) and the gauge string fields should be tensored with $\sigma_{2,3}$. The operators Q and η_0 are tensored with σ_3 . The action then should be supplemented with a factor of $\frac{1}{2}$ and a trace over this “internal Chan-Paton” space, in order to obtain the usual result for the NS+ sector. The various sectors should be put into the string field, presumably tensored also with genuine Chan-Paton factors, according to the details of the D-brane system one wants to describe.

The generalization of these ideas to the cubic theory is simple [68]. Here, the parity of the string fields and the gauge string fields is reversed. Hence, the string fields come with $\sigma_{2,3}$ factors and the gauge string fields get the $\sigma_{0,1}$. The Y_{-2} insertion in the action comes with a σ_3 factor, just like Q . No other changes are necessary. The case at hand is nothing but a variant of this theory. Thus, the inclusion of the NS− sector should be done exactly in the same way, only, as we explained in section 3, the GSO projection should be performed over the minimal and non-minimal sectors together. The sole difference is that the Y_{-2} insertion is now replaced by the \mathcal{N}_ρ insertion. Hence, all will work well provided we assign a σ_3 factor also to \mathcal{N}_ρ .

In the minimal theory the Ramond sector can (naively) be included by introducing a picture $-\frac{1}{2}$ Ramond string field and adding to the action the following term [6],

$$S_R = - \int Y \left(\frac{1}{2} \alpha Q \alpha + A \alpha^2 \right). \tag{6.1}$$

Here, α is the Ramond string field, residing in the R+ or R− sector, according to the D-brane at hand. Since the R+, R− and NS− sectors are mutually exclusive, there can be no need for adding the internal Chan-Paton factors when the Ramond sector is considered. One might have thought that the Ramond part of the action should be written as,

$$S_R \stackrel{?}{=} - \int \mathcal{N}_\rho \left(\frac{1}{2} \alpha Q \alpha + A \alpha^2 \right), \tag{6.2}$$

where the \mathcal{N}_ρ factor is inserted at the mid-point. This cannot be the case, since only one of the zero modes of u is addressed in this way. Another possibility could have been to use the same insertion as in the NS sector, i.e., \mathcal{N}_ρ , while putting on top of it also one picture changing operator X . This, however, would be inconsistent due to singularities in the OPE of X and $[Q, \chi]$. Redefining the picture of α to be $+\frac{1}{2}$ and plugging Y instead of X does not lead to immediate singularities, but those would nevertheless emerge after the integration of u . Hence, one should look for a less trivial modification of the insertion in order to define the Ramond sector. We shall not try that, since even upon establishing a sensible insertion for the Ramond sector, its gauge transformation would not be well defined. This would be just the same as for the minimal theory, in which collisions of picture changing operators

occur upon iterating the fermionic part of the gauge symmetry [11]. This destroys the finite gauge symmetry, which we should have if the theory is to describe superstring theory. We cannot expect a non-minimal field to replace the X insertion in the gauge transformation, since a genuine picture changing should be enforced in order to obtain a string field in the picture that we specified upon defining the theory. Thus, it seems that the non-minimal theory cannot resolve the problems with the Ramond sector. Another resolution is needed. Some preliminary ideas regarding such a resolution were presented in [11].

7 Conclusions

This work focused on two complementary issues, the definition of the space of string fields and the definition of the (super)string field theory. While we still do not know how to properly define the space of string fields, we made some observations regarding its properties. We hope that these observations would be useful for properly defining this elusive space.

The observations we made regarding the space of string fields enabled us to conclude that the common obstruction to the Y_{-2} insertion in the standard cubic superstring field theory is not well founded. These observations also enabled us to recognize a problem (non-zero conformal weight of a mid-point insertion) with the definition of the alternative formalism [12], devised by Berkovits and Siegel in order to avoid the Y_{-2} insertion. We managed to show that the non-minimal theory can be modified in a way that avoids the problems with the conformal weight.

Nonetheless, as we showed in the companion paper [11], there is a genuine problem with the minimal formulation. The origin of this problem is not the kernel of Y_{-2} and its role in defining the action, but rather the singularity of OPE's of picture changing operators and their presence in defining finite gauge transformations. Unfortunately, it seems that the non-minimal theory cannot resolve these issues. It would be highly desirable to have a (cubic) theory that can be defined with string fields with only interior-insertions and whose gauge transformations do not contain explicit operator insertions. Such a theory would be well defined in both sectors presumably even at the quantum level. We are currently studying this possibility.

During our study of the non-minimal theory we verified that the vacuum solution on a BPS manifold exists also in this theory. We interpreted that as an evidence in favour of the validity of the non-minimal formalism. However, one might as well interpret this observation as standing in favour of the validity of this solution. Other than by its action and the cohomology around it, a solution can also be characterized by the boundary state to which it corresponds [69].²⁵ It would be interesting to extend this construction also to the various superstring field theories. This would further support our interpretation of the superstring vacuum solution.

²⁵See also [41, 70–75].

Acknowledgments

I would like to thank Nathan Berkovits, Ted Erler, Udi Fuchs, Michael Kiermaier, Leonardo Rastelli and Barton Zwiebach for many discussions on the issues covered in this work.

It is a pleasure to thank the organizers and participants of the KITP workshop “Fundamental Aspects of Superstring Theory”, where a major part of this work was performed, for providing a very stimulating and enjoyable environment. While at the KITP, this research was supported by the National Science Foundation under Grant No. PHY05-51164. It is likewise a pleasure to thank the Simons Center for Geometry and Physics and the organizers and participants of the “Simons Workshop on String Field Theory” for a great hospitality and for many discussions on and around the topics presented in this manuscript.

This work is supported by the U.S. Department of Energy (D.O.E.) under cooperative research agreement DE-FG0205ER41360. My research is supported by an Outgoing International Marie Curie Fellowship of the European Community. The views presented in this work are those of the author and do not necessarily reflect those of the European Community.

References

- [1] E. Witten, *Interacting field theory of open superstrings*, *Nucl. Phys. B* **276** (1986) 291 [[SPIRES](#)].
- [2] E. Witten, *Noncommutative geometry and string field theory*, *Nucl. Phys. B* **268** (1986) 253 [[SPIRES](#)].
- [3] E. Fuchs and M. Kroyter, *Analytical solutions of open string field theory*, [arXiv:0807.4722](#) [[SPIRES](#)].
- [4] D. Friedan, E.J. Martinec and S.H. Shenker, *Conformal invariance, supersymmetry and string theory*, *Nucl. Phys. B* **271** (1986) 93 [[SPIRES](#)].
- [5] C. Wendt, *Scattering amplitudes and contact interactions in Witten’s superstring field theory*, *Nucl. Phys. B* **314** (1989) 209 [[SPIRES](#)].
- [6] C.R. Preitschopf, C.B. Thorn and S.A. Yost, *Superstring field theory*, *Nucl. Phys. B* **337** (1990) 363 [[SPIRES](#)].
- [7] I.Y. Arefeva, P.B. Medvedev and A.P. Zubarev, *Background formalism for superstring field theory*, *Phys. Lett. B* **240** (1990) 356 [[SPIRES](#)].
- [8] I.Y. Arefeva, P.B. Medvedev and A.P. Zubarev, *New representation for string field solves the consistency problem for open superstring field theory*, *Nucl. Phys. B* **341** (1990) 464 [[SPIRES](#)].
- [9] N. Berkovits, *SuperPoincaré invariant superstring field theory*, *Nucl. Phys. B* **450** (1995) 90 [[hep-th/9503099](#)] [[SPIRES](#)].
- [10] E. Fuchs and M. Kroyter, *On the classical equivalence of superstring field theories*, *JHEP* **10** (2008) 054 [[arXiv:0805.4386](#)] [[SPIRES](#)].
- [11] M. Kroyter, *Superstring field theory equivalence: Ramond sector*, [arXiv:0905.1168](#) [[SPIRES](#)].
- [12] N. Berkovits and W. Siegel, *Regularizing cubic open Neveu-Schwarz string field theory*, [arXiv:0901.3386](#) [[SPIRES](#)].

- [13] M. Henneaux and C. Teitelboim, *Quantization of gauge systems*, Princeton University Press, Princeton U.S.A. (1992), pag. 520.
- [14] N. Berkovits, *Pure spinor formalism as an $N = 2$ topological string*, *JHEP* **10** (2005) 089 [[hep-th/0509120](#)] [[SPIRES](#)].
- [15] M. Schnabl, *Analytic solution for tachyon condensation in open string field theory*, *Adv. Theor. Math. Phys.* **10** (2006) 433 [[hep-th/0511286](#)] [[SPIRES](#)].
- [16] Y. Okawa, *Comments on Schnabl's analytic solution for tachyon condensation in Witten's open string field theory*, *JHEP* **04** (2006) 055 [[hep-th/0603159](#)] [[SPIRES](#)].
- [17] E. Fuchs and M. Kroyter, *On the validity of the solution of string field theory*, *JHEP* **05** (2006) 006 [[hep-th/0603195](#)] [[SPIRES](#)].
- [18] A. Sen, *Descent relations among bosonic D-branes*, *Int. J. Mod. Phys. A* **14** (1999) 4061 [[hep-th/9902105](#)] [[SPIRES](#)].
- [19] A. Sen, *Universality of the tachyon potential*, *JHEP* **12** (1999) 027 [[hep-th/9911116](#)] [[SPIRES](#)].
- [20] I. Ellwood and M. Schnabl, *Proof of vanishing cohomology at the tachyon vacuum*, *JHEP* **02** (2007) 096 [[hep-th/0606142](#)] [[SPIRES](#)].
- [21] I. Ellwood and W. Taylor, *Open string field theory without open strings*, *Phys. Lett. B* **512** (2001) 181 [[hep-th/0103085](#)] [[SPIRES](#)].
- [22] B. Feng, Y.-H. He and N. Moeller, *Testing the uniqueness of the open bosonic string field theory vacuum*, [hep-th/0103103](#) [[SPIRES](#)].
- [23] M. Bochicchio, *Gauge fixing for the field theory of the bosonic string*, *Phys. Lett. B* **193** (1987) 31 [[SPIRES](#)].
- [24] M. Bochicchio, *String field theory in the Siegel gauge*, *Phys. Lett. B* **188** (1987) 330 [[SPIRES](#)].
- [25] C.B. Thorn, *Perturbation theory for quantized string fields*, *Nucl. Phys. B* **287** (1987) 61 [[SPIRES](#)].
- [26] L. Rastelli, *Comments on the open string C^* algebra*, talk presented at the *Simons Workshop on String Field Theory*, Simons Center, march 23–27, Stony Brook University, Stony Brook, U.S.A. (2009).
- [27] T. Takahashi and S. Tanimoto, *Wilson lines and classical solutions in cubic open string field theory*, *Prog. Theor. Phys.* **106** (2001) 863 [[hep-th/0107046](#)] [[SPIRES](#)].
- [28] J. Kluson, *Exact solutions of open bosonic string field theory*, *JHEP* **04** (2002) 043 [[hep-th/0202045](#)] [[SPIRES](#)].
- [29] T. Takahashi and S. Tanimoto, *Marginal and scalar solutions in cubic open string field theory*, *JHEP* **03** (2002) 033 [[hep-th/0202133](#)] [[SPIRES](#)].
- [30] N. Drukker, *On different actions for the vacuum of bosonic string field theory*, *JHEP* **08** (2003) 017 [[hep-th/0301079](#)] [[SPIRES](#)].
- [31] I. Kishimoto and T. Takahashi, *Vacuum structure around identity based solutions*, [arXiv:0904.1095](#) [[SPIRES](#)].
- [32] L. Rastelli and B. Zwiebach, *Tachyon potentials, star products and universality*, *JHEP* **09** (2001) 038 [[hep-th/0006240](#)] [[SPIRES](#)].
- [33] G.T. Horowitz, J.D. Lykken, R. Rohm and A. Strominger, *A purely cubic action for string field theory*, *Phys. Rev. Lett.* **57** (1986) 283 [[SPIRES](#)].

- [34] G.T. Horowitz and A. Strominger, *Translations as inner derivations and associativity anomalies in open string field theory*, *Phys. Lett. B* **185** (1987) 45 [SPIRES].
- [35] I. Bars and Y. Matsuo, *Associativity anomaly in string field theory*, *Phys. Rev. D* **65** (2002) 126006 [hep-th/0202030] [SPIRES].
- [36] K. Furuuchi and K. Okuyama, *Comma vertex and string field algebra*, *JHEP* **09** (2001) 035 [hep-th/0107101] [SPIRES].
- [37] L. Rastelli, A. Sen and B. Zwiebach, *Boundary CFT construction of D-branes in vacuum string field theory*, *JHEP* **11** (2001) 045 [hep-th/0105168] [SPIRES].
- [38] M. Schnabl, *Wedge states in string field theory*, *JHEP* **01** (2003) 004 [hep-th/0201095] [SPIRES].
- [39] L. Rastelli, A. Sen and B. Zwiebach, *Vacuum string field theory*, hep-th/0106010 [SPIRES].
- [40] H. Hata and T. Kawano, *Open string states around a classical solution in vacuum string field theory*, *JHEP* **11** (2001) 038 [hep-th/0108150] [SPIRES].
- [41] D. Gaiotto, L. Rastelli, A. Sen and B. Zwiebach, *Ghost structure and closed strings in vacuum string field theory*, *Adv. Theor. Math. Phys.* **6** (2003) 403 [hep-th/0111129] [SPIRES].
- [42] A. LeClair, M.E. Peskin and C.R. Preitschopf, *String field theory on the conformal plane. 1. Kinematical principles*, *Nucl. Phys. B* **317** (1989) 411 [SPIRES].
- [43] A. LeClair, M.E. Peskin and C.R. Preitschopf, *String field theory on the conformal plane. 2. Generalized gluing*, *Nucl. Phys. B* **317** (1989) 464 [SPIRES].
- [44] M. Schnabl, *Anomalous reparametrizations and butterfly states in string field theory*, *Nucl. Phys. B* **649** (2003) 101 [hep-th/0202139] [SPIRES].
- [45] D. Gaiotto, L. Rastelli, A. Sen and B. Zwiebach, *Star algebra projectors*, *JHEP* **04** (2002) 060 [hep-th/0202151] [SPIRES].
- [46] E. Fuchs, M. Kroyter and A. Marcus, *Squeezed state projectors in string field theory*, *JHEP* **09** (2002) 022 [hep-th/0207001] [SPIRES].
- [47] S. Uhlmann, *A note on kappa-diagonal surface states*, *JHEP* **11** (2004) 003 [hep-th/0408245] [SPIRES].
- [48] E. Fuchs and M. Kroyter, *On surface states and star-subalgebras in string field theory*, *JHEP* **10** (2004) 004 [hep-th/0409020] [SPIRES].
- [49] E. Fuchs and M. Kroyter, *Normalization anomalies in level truncation calculations*, *JHEP* **12** (2005) 031 [hep-th/0508010] [SPIRES].
- [50] A. Belopolsky, *Picture changing operators in supergeometry and superstring theory*, hep-th/9706033 [SPIRES].
- [51] L. Rastelli and B. Zwiebach, *Solving open string field theory with special projectors*, *JHEP* **01** (2008) 020 [hep-th/0606131] [SPIRES].
- [52] Y. Okawa, L. Rastelli and B. Zwiebach, *Analytic solutions for tachyon condensation with general projectors*, hep-th/0611110 [SPIRES].
- [53] T. Erler, *Tachyon vacuum in cubic superstring field theory*, *JHEP* **01** (2008) 013 [arXiv:0707.4591] [SPIRES].
- [54] I.Y. Aref'eva, R.V. Gorbachev and P.B. Medvedev, *Tachyon solution in cubic Neveu-Schwarz string field theory*, *Theor. Math. Phys.* **158** (2009) 320 [arXiv:0804.2017] [SPIRES].
- [55] T. Erler, *Split string formalism and the closed string vacuum*, *JHEP* **05** (2007) 083 [hep-th/0611200] [SPIRES].

- [56] T. Erler, *Split string formalism and the closed string vacuum. II*, *JHEP* **05** (2007) 084 [[hep-th/0612050](#)] [[SPIRES](#)].
- [57] M. Schnabl, *Comments on marginal deformations in open string field theory*, *Phys. Lett. B* **654** (2007) 194 [[hep-th/0701248](#)] [[SPIRES](#)].
- [58] M. Kiermaier, Y. Okawa, L. Rastelli and B. Zwiebach, *Analytic solutions for marginal deformations in open string field theory*, *JHEP* **01** (2008) 028 [[hep-th/0701249](#)] [[SPIRES](#)].
- [59] T. Erler, *Marginal solutions for the superstring*, *JHEP* **07** (2007) 050 [[arXiv:0704.0930](#)] [[SPIRES](#)].
- [60] Y. Okawa, *Analytic solutions for marginal deformations in open superstring field theory*, *JHEP* **09** (2007) 084 [[arXiv:0704.0936](#)] [[SPIRES](#)].
- [61] Y. Okawa, *Real analytic solutions for marginal deformations in open superstring field theory*, *JHEP* **09** (2007) 082 [[arXiv:0704.3612](#)] [[SPIRES](#)].
- [62] E. Fuchs, M. Kroyter and R. Potting, *Marginal deformations in string field theory*, *JHEP* **09** (2007) 101 [[arXiv:0704.2222](#)] [[SPIRES](#)].
- [63] E. Fuchs and M. Kroyter, *Marginal deformation for the photon in superstring field theory*, *JHEP* **11** (2007) 005 [[arXiv:0706.0717](#)] [[SPIRES](#)].
- [64] M. Kiermaier and Y. Okawa, *Exact marginality in open string field theory: a general framework*, [arXiv:0707.4472](#) [[SPIRES](#)].
- [65] M. Kiermaier and Y. Okawa, *General marginal deformations in open superstring field theory*, [arXiv:0708.3394](#) [[SPIRES](#)].
- [66] N. Berkovits, *The tachyon potential in open Neveu-Schwarz string field theory*, *JHEP* **04** (2000) 022 [[hep-th/0001084](#)] [[SPIRES](#)].
- [67] N. Berkovits, A. Sen and B. Zwiebach, *Tachyon condensation in superstring field theory*, *Nucl. Phys. B* **587** (2000) 147 [[hep-th/0002211](#)] [[SPIRES](#)].
- [68] I.Y. Arefeva, D.M. Belov and A.A. Giryavets, *Construction of the vacuum string field theory on a non-BPS brane*, *JHEP* **09** (2002) 050 [[hep-th/0201197](#)] [[SPIRES](#)].
- [69] M. Kiermaier, Y. Okawa and B. Zwiebach, *The boundary state from open string fields*, [arXiv:0810.1737](#) [[SPIRES](#)].
- [70] A. Hashimoto and N. Itzhaki, *Observables of string field theory*, *JHEP* **01** (2002) 028 [[hep-th/0111092](#)] [[SPIRES](#)].
- [71] Y. Michishita, *On-shell gauge invariants and field strengths in open superstring field theory*, *Nucl. Phys. B* **698** (2004) 111 [[hep-th/0406242](#)] [[SPIRES](#)].
- [72] I. Ellwood, *The closed string tadpole in open string field theory*, *JHEP* **08** (2008) 063 [[arXiv:0804.1131](#)] [[SPIRES](#)].
- [73] T. Kawano, I. Kishimoto and T. Takahashi, *Gauge invariant overlaps for classical solutions in open string field theory*, *Nucl. Phys. B* **803** (2008) 135 [[arXiv:0804.1541](#)] [[SPIRES](#)].
- [74] T. Kawano, I. Kishimoto and T. Takahashi, *Schnabl's solution and boundary states in open string field theory*, *Phys. Lett. B* **669** (2008) 357 [[arXiv:0804.4414](#)] [[SPIRES](#)].
- [75] I. Kishimoto and T. Takahashi, *Numerical evaluation of gauge invariants for a-gauge solutions in open string field theory*, *Prog. Theor. Phys.* **121** (2009) 695 [[arXiv:0902.0445](#)] [[SPIRES](#)].